# Notes On Gain-Error In Op-Amp Amplifiers

This article is about the errors you can make in calculating the gain of an op-amp amplifier circuit. I'm assuming here that you are familiar with op-amp amplifier circuits. But let's do a quick review anyway.

As you know, the key idea in op-amp circuits is that you start with a very high gain, and then trade off that gain in exchange for increased bandwidth and improved characteristics. What characteristics? You remember; things like input impedance (it gets bigger), output impedance (it gets smaller), distortion (it becomes less), and so forth.

Op-amps have enormous open-loop gain $(A_{ol}$. Open-loop gain is the gain of the op-amp chip itself with no feedback. That gain is too big to be used, so you lower it with negative feedback. The gain with feedback is the closed-loop gain $(A_{cl}$.

Below are schematics for the two basic feedback circuits: the inverting amplifier and the non-inverting amplifier. The gain equation for each circuit is included. Notice that the gain equations do not include frequency as a variable.

Inverting Amplifier

$$A_v = - \frac{R_f}{R_i}$$

Non-Inverting Amplifier

$$A_v = 1 + \frac{R1}{R2}$$
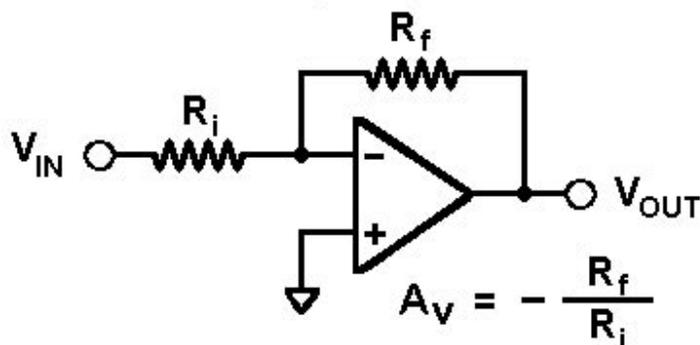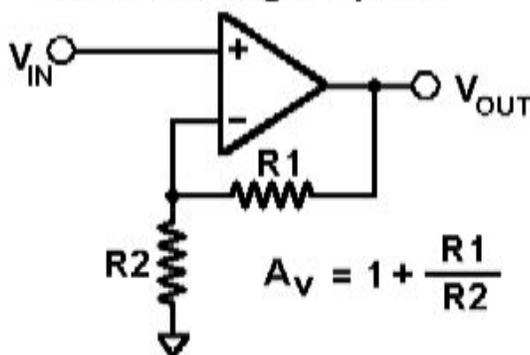
Before we get to the punch-line of this article, there's a short story to tell. So, please be patient.

Many books either say or imply that the closed-loop gain doesn't change with frequency until the line for ACL meets the line for AOL on the amplifier's Bode plot. What's a Bode plot? C'mon, you remember! It's a graph that shows how the gain of an amplifier "rolls off" as signal frequency increases. Many op-amps, like the lovable old 741, roll off at 20 dB per decade. (A decade is when the frequency changes by a factor of 10, but you knew that.) The open-loop gain of an op-amp starts rolling off at a relatively low frequency, maybe 10 Hertz. But they have so much AOL that it doesn't get to 1 (0 dB) until you get up to mega-Hertz.

Hey! Someone left a Bode plot right here for us to look at! It could be for a 741.
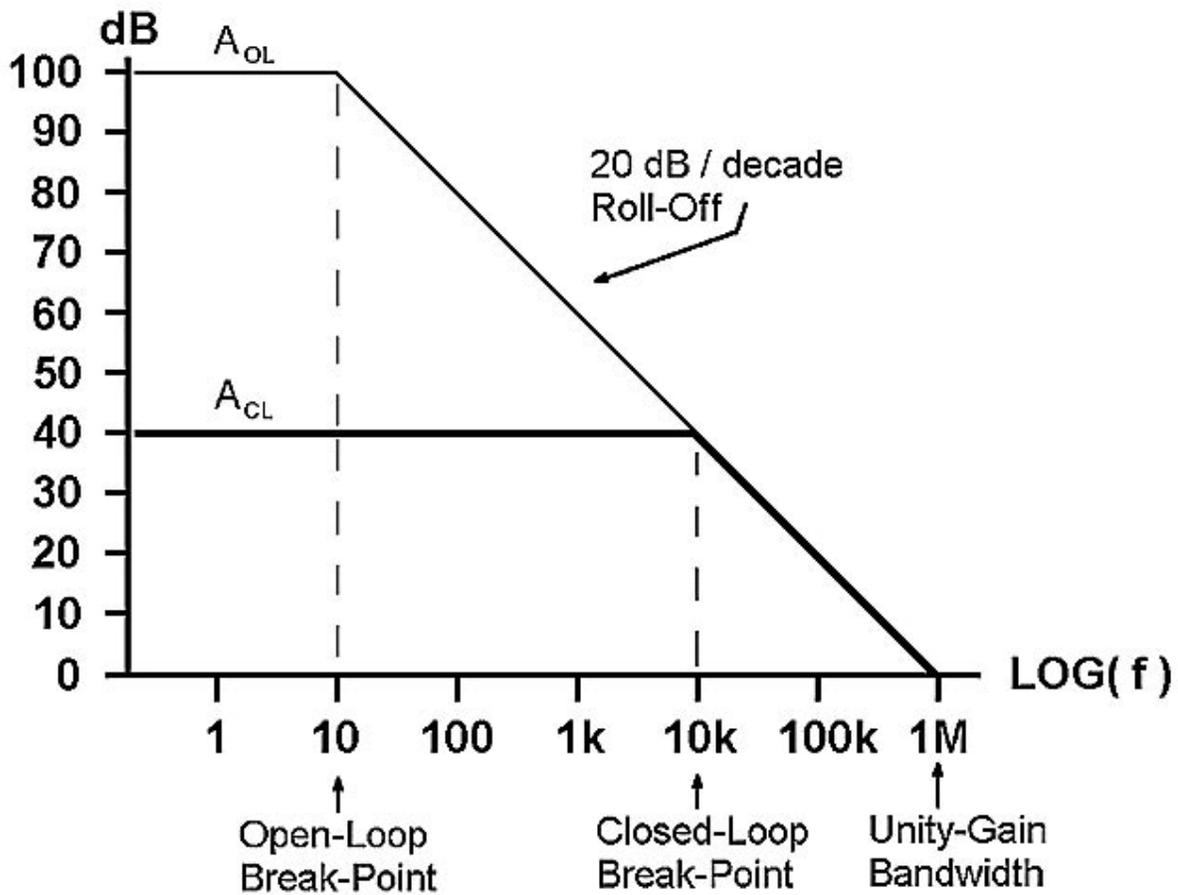
OK, you've been patient. Here's the punch-line: ACL does NOT stay constant until it hits the roll-off. A decade before the roll-off, when AOL is still 20 dB higher than ACL, you've already lost about 10% of your closed-loop gain!

What? You're shocked? You don't believe me? I can understand. But remember, it's not the things you don't know that get you into trouble. Instead, it's the things you do know, but which turn out to be wrong. But it's always good to be skeptical, so the math is below. Better yet, build a circuit and measure the closed loop gain as you get close to the roll-off and see if the gain stays constant or not.

Ideal closed-loop gain value is $\quad A_{cl} = \dfrac{1}{\beta} \quad$ where β is the feedback ratio

Actual closed-loop gain value is $\quad A_{cl} = \dfrac{A_{ol}}{1 + \beta A_{ol}}$

Let's call the ideal closed loop gain value A_v

We can express the difference between the ideal value and the actual value as $\Delta = A_v - A_{cl}$

The difference as a fraction of the ideal closed-loop gain is $\quad \delta = \dfrac{\Delta}{A_v}$

which we can calculate as $\delta = \dfrac{A_v - A_{v_1}}{A_v} = 1 - \dfrac{A_{c_1}}{A_v} = 1 - \dfrac{\frac{A_\alpha}{1 + \beta A_\alpha}}{A_v}$

Let $A_{\alpha_1} = N \times A_c$ meaning that the open-loop gain is N times bigger than A_v now we have

$$\delta = 1 - \dfrac{\frac{N A_v}{1 + \beta N A_v}}{A_v} = 1 - \dfrac{N}{1 + \beta A_v N}$$

But, with an "ideal" op-amp, the closed-loop gain is $A_v = \dfrac{1}{\beta}$

so

$$\delta = 1 - \dfrac{N}{1 + N} = \dfrac{1}{1 + N}$$

If the open-loop gain is 20 dB more than the closed-loop gain then N = 10 which gives

$$\delta = \dfrac{1}{1 + 10} = \dfrac{1}{11} = 0.091$$

or an error of 9.1%

An error of 9.1% is not negligible.